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OnMeromorphicSolutions ofaCertainType ofSystemofComplex Differential –Difference equations

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Abstract. In this article, we study meromorphic solutions of the type of system of complex differential and difference equation of the following form

$$\sum_{j=1}^{n}\left[\alpha_{1j}(z)f_1^{(\lambda_{1j})}(z+c_j)+\beta_{1j}(z)f_1^{m}(z)\right]=R_2(z,f_2(z)),$$
$$\sum_{j=1}^{n}\left[\alpha_{2j}(z)f_2^{(\lambda_{2j})}(z+c_j)+\beta_{2j}(z)f_2^{m}(z)\right]=R_1(z,f_1(z)).$$

Where λ_{ij} (j=1,2,...n; i=1,2) are finite non-negative integers and c_j (j=1,2,...n) are distinct, nonzero complex numbers. $\alpha_{ij}(z)$, $\beta_{ij}(z)$ (j=1,2,...n) are small functions relative to f_i(z) (i=1,2) respectively, R_i (z, f(z)) (i=1,2) are rational in f_i (z) (i=1,2) with coefficients which are small functions of f_i (z) (i=1,2) respectively.

Key words and phrases. Nevanlinna theory; differential-difference equations; meromorphic functions.

1. INTRODUCTION

In this article, we assume that the reader is familiar with standard symbols and fundamental results of Nevanlinnna theory. A differential polynomial of f(z) means that it is a polynomial in f(z) and its derivatives with small functions of f(z) as coefficients. A differential-difference polynomial of f(z) means that it is a polynomial in f(z), its derivatives and its shifts f(z + c) with small functions of f(z) as coefficients. A meromorphic function a(z) is called a small functions of f(z) if and only if T (r, a(z)) = S(r, f). Let c be a fixed, non-zero complex number Δ_cf(z) = f(z +c) - f(z) and Δ_c^nf(z) = Δ_c (Δ_c^{n-1}f(z)) = Δ_c^{n-1}f(z +c) - Δ_c^{n-1}f(z) for each integer n$\geq$ 2. In order to simplify our notation, we shall use the same notationΔ for both a general c and when c = 1. Equations written with the above difference operations Δ_c^nf(z) are difference equations.

In 2000, Ablowitz[1] proved some results on the classical Malmquist type theorem of the complexdifference equations by applying Nevanlinna theory. They obtained a typical results as follows.

Theorem A. If a complex difference equation

$$f(z+1)+f(z-1)=R(z,f(z))=\frac{a_0(z)+a_1(z)f(z)+\cdots+a_p(z)f(z)^p}{b_0(z)+b_1(z)f(z)+\cdots+b_p(z)f(z)^q}$$

with polynomial coefficients a_i (i = 1, 2, ... , p) and a_j (j = 1, 2, ... , q), admits a transcendental meromorphic solution of finite order, then d = max{p,q} $\leq$ 2.

One year later, Heittokangas[5] extended and improved the above result to the case of higher-order difference equations of more general type. They got the following result.

Theorem B. Let $c_1, c_2, \ldots, c_n \in C \setminus \{0\}$. If the difference equation

$$\sum_{i=1}^{n} f(z+c_i)=R(z,f(z))=\frac{a_0(z)+a_1(z)f(z)+\cdots+a_p(z)f(z)^p}{b_0(z)+b_1(z)f(z)+\cdots+b_p(z)f(z)^q}$$

with polynomial coefficients a_i(i = 1, 2, ... , p) and b_j(j = 1, 2, ... , q), admits atranscendental meromorphic solution of finite order, then d = max{p,q} $\leq$ n.

In [8], Laine et al. obtained Tumura-Clunie theorem about the complex differenceequation of the form

$$\sum_{i=1}^{n} \alpha_i(z) f(z + c_i) = R(z, f(z)) = \frac{P(z, f(z))}{Q(z, f(z))},$$

where the coefficients $\alpha_i(z)$ (i = 1, 2,..., n) are non-vanishing small functions relative to f(z) and P(z, f(z)), Q(z, f(z)) are relating prime polynomials in f(z) over the field of small functions relative to f(z).

Theorem C. Suppose that $c_1, c_2, \ldots, c_n$ are distinct, nonzero complex numbers and that f(z) is a transcendental meromorphic solution of the last difference equation. Moreover, we assume that q $= deg_f Q > 0$,

$$n = \max\{p, q\} = \max\{deg_f P, deg_f Q\},$$

and that, without restricting generality, Q is a monic polynomial. If there exist a∈ [0,n) such that for all r sufficiently large,

$$\bar{N}\left(r, \sum_{i=1}^{n} \alpha_i(z) f(z + c_i)\right) \leq \alpha \bar{N}(r + C, f(z)) + S(r, f),$$

whereC = $\max\{|c_1|, |c_2|, \ldots |c_n|\}$, then either the order $\rho(f) = +\infty$, or

$$Q(z, f(z)) = (f(z) + h(z))^q,$$

where h(z) is a small meromorphic function relative to f(z).

In 2015, Haichou LI and LingyunGao[6], proved the following results, whichgeneralize the above related results to systems of complex differential and differenceequations.

Theorem D. Let $c_1, c_2, \ldots, c_n$ be distinct, nonzero complex numbers and supposethat ($f_1(z)$, $f_2(z)$) is a transcendental meromorphic solution of the following system

$$\sum_{j=1}^{n} \alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) = R_2(z, f_2(z)) = \frac{a_{10}(z) + a_{11}(z) f_2(z) + \cdots + a_{1p_2}(z) f_2(z)^{p_2}}{b_{10}(z) + b_{11}(z) f_2(z) + \cdots + b_{1q_2}(z) f_2(z)^{q_2}},$$

$$\sum_{j=1}^{n} \alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) = R_1(z, f_1(z)) = \frac{a_{20}(z) + a_{21}(z) f_1(z) + \cdots + a_{2p_1}(z) f_1(z)^{p_1}}{b_{20}(z) + b_{21}(z) f_1(z) + \cdots + b_{2q_1}(z) f_1(z)^{q_1}}$$

with coefficients $\alpha_{ij}(z)(i = 1, 2;\ j = 1, 2, \ldots, n)$, $a_{ij}(j = 1, 2, \ldots, p_i;\ i = 1, 2)$ and $b_{ij}(j = 1, 2, \ldots, q;\ i = 1, 2)$ are small functions relative to $f_i(z)$(i = 1, 2), respectively, where λ_{ij} (j = 1, 2, . . . , n; i = 1, 2) are finite non-negative integers, and denote

$$d_i = deg_{f_i} R_i(z, f_i(z)) = \max\{p_i, q_i\};$$

$$\lambda_i = \sum_{j=1}^{n} (\lambda_{ij} + 1), i = 1, 2.$$

If the order $\rho(f_i)$ (i = 1, 2) are finite, then $d_1 d_2 \leq \lambda_1 \lambda_2$.

Theorem E. Let ($f_1(z)$, $f_2(z)$) be a transcendental meromorphic solution of thefollowingsystem

$$\sum_{j=1}^{n} \alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) = R_2(z, f_2(z)) = \frac{P_2(z, f_2(z))}{Q_2(z, f_2(z))},$$

$$\sum_{j=1}^{n} \alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) = R_1(z, f_1(z)) = \frac{P_1(z, f_1(z))}{Q_1(z, f_1(z))}$$

where λ_{ij} (j = 1, 2,..., n; i = 1, 2) are finite non-negative integers, and c_j (j = 1, 2,..., n) are distinct, nonzero complex numbers, $\alpha_{ij}(z)$(i = 1, 2; j = 1, 2,..., n) are small functions relative to $f_i(z)$ (i = 1, 2) respectively. $P_i(z, f_i(z))$, $Q_i(z, f_i(z))$ are relating prime polynomials in $f_i(z)$ over the field of small functions relative to $f_i(z)$ (i=1, 2), denote $n_i = \max\{p_i, q_i\} = \max\{deg_{f_i} P_i, deg_{f_i} Q_i\}$, $\lambda_i = \sum_{j=1}^{n} (\lambda_{ij} + 1)$(i = 1, 2). Moreover, we assume that $q_i = deg_{f_i} Q_i > 0$ (i = 1, 2), and that, without restricting generality, Q_i (i = 1, 2) are both monic polynomials. If there exist $\beta_i \epsilon [0, n_i)$(i = 1, 2) such that for all r sufficiently large,

$$\bar{N}\left(r, \sum_{j=1}^{n} \alpha_{ij} f_i^{(\lambda_{ij})}(z + c_j)\right) \leq \beta_i \bar{N}(r + C, f_i(z)) + S(r, f_i)$$

$(i = 1,2)$, where C = $\max\{|c_1|, |c_2|, \ldots |c_n|\}$.

Then either at least one of the orders $\rho(f_i) = \infty$, (i = 1, 2)is true; or

$$Q_i(z, f_i(z)) = (f_i(z) + h_i(z))^{q_i} (i = 1, 2),$$

where $h_i(z)$ (i = 1, 2)are small meromorphic functions relative to $f_i(z)$ (i = 1, 2), respectively.

Theorem F. Let c_1, c_2,...,c_n be distinct, nonzero complex numbers, and suppose $(f_1(z), f_2(z))$ is a transcendental meromorphic solution of the system of differential and difference equations

$$\sum_{j=1}^{n} \alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) = f_2(P(z)),$$

$$\sum_{j=1}^{n} \alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) = f_1(P(z))$$

where λ_{ij} (j = 1, 2,..., n; i = 1, 2) are finite non-negative integers, and α_{ij} (i = 1, 2; j = 1, 2,..., n) are small functions relative to $f_i(z)$ (i = 1, 2) respectively, $\lambda_i = \sum_{j=1}^{n} (\lambda_{ij} + 1)$ (i = 1, 2), p(z) is a polynomial of degree $k \geq 2$ and that $\lambda_i \geq k$. Then, given $\varepsilon > 0$, we have
$T(r, f_1) + T(r, f_2) = O((\log r))^{\alpha+\varepsilon}$,
where $\alpha = \frac{\log \lambda}{\log k}$, $\lambda = \max\{\lambda_1, \lambda_2\}$.

In this paper, we consider the type of system of complex differential-difference equations of the following form

$$\sum_{j=1}^{n} [\alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) + \beta_{1j}(z) f_1^m(z)] = R_2(z, f_2(z))$$

$$\sum_{j=1}^{n} [\alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) + \beta_{2j}(z) f_2^m(z)] = R_1(z, f_1(z))$$

where λ_{ij} (j = 1, 2,..., n; i = 1, 2) are finite non-negative integers, and c_j (j = 1, 2,..., n) are distinct, nonzero complex numbers, $\alpha_{ij}(z), \beta_{ij}(z)$(i = 1, 2; j = 1, 2,..., n) are small functions relative to $f_i(z)$ (i = 1, 2) respectively, $R_i(z, f_i(z))$, (i=1,2) are rational in $f_i(z)$(i=1,2) respectively.

Here we obtain the following result, which generalize the above related results to systems of complex differential and difference equations.

Theorem 1.1. Let c_1, c_2,...,c_n be distinct, nonzero complex numbers and suppose that $(f_1(z), f_2(z))$ is a transcendental meromorphic solution of the following system

(1.1)
$$\sum_{j=1}^{n} [\alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) + \beta_{1j}(z) f_1^m(z)] = R_2(z, f_2(z)) = \frac{a_{10}(z) + a_{11}(z) f_2(z) + \cdots + a_{1p_2}(z) f_2(z)^{p_2}}{b_{10}(z) + b_{11}(z) f_2(z) + \cdots + b_{1q_2}(z) f_2(z)^{q_2}},$$

(1.2)

$$\sum_{j=1}^{n} [\alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) + \beta_{2j}(z) f_2^m(z)] = R_1(z, f_1(z)) = \frac{a_{20}(z) + a_{21}(z) f_1(z) + \cdots + a_{2p_1}(z) f_1(z)^{p_1}}{b_{20}(z) + b_{21}(z) f_1(z) + \cdots + b_{2q_1}(z) f_1(z)^{q_1}}$$

with coefficients $\alpha_{ij}(z), \beta_{ij}(z)$ (i = 1, 2; j = 1, 2, ..., n), a_{ij} (j = 1,2, ..., p_i; i = 1,2) and b_{ij} (j = 1,2,.., q; i = 1,2) are small functions relative to $f_i(z)$(i = 1, 2), respectively, where λ_{ij} (j = 1, 2, ..., n; i = 1, 2) are finite non-negative integers, and denote

$$d_i = deg_{f_i} R_i(z, f_i(z)) = \max\{p_i, q_i\};$$

$$\lambda_i = \sum_{j=1}^{n} (\lambda_{ij} + 1), i = 1,2.$$

If the order $\rho(f_i)$ (i = 1, 2) are finite, then

$$d_1 d_2 \leq (\lambda_1 + m)(\lambda_2 + m).$$

Theorem 1.2. Let c_1, c_2,...,c_n be distinct, nonzero complex numbers, and suppose $(f_1(z), f_2(z))$ is a transcendental meromorphic solution of the system of differential and difference equations

(1.3)
$$\sum_{j=1}^{n} [\alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) + \beta_{1j}(z) f_1^m(z)] = f_2(P(z)),$$

(1.4)

$$\sum_{j=1}^{n} [\alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) + \beta_{2j}(z) f_2^m(z)] = f_1(P(z)),$$

whereλ_{ij} (j = 1, 2,..., n; i = 1, 2) are finite non-negative integers, and $\alpha_{ij}(z), \beta_{ij}(z)$(i = 1, 2; j = 1, 2,..., n) are small functions relative to $f_i(z)$ (i = 1, 2) respectively, denote $\lambda_i = \sum_{j=1}^{n} (\lambda_{ij} + 1)$ (i = 1, 2), p(z) is a polynomial of degree k ≥ 2 and that $\lambda_i \geq k$. Then, given $\varepsilon > 0$, we have

$$T(r, f_1) + T(r, f_2) = O((\log r))^{\alpha+\varepsilon},$$

where$\frac{\log \lambda}{\log k}$, $\lambda = \max\{\lambda_1, \lambda_2\}$.

2. SOME LEMMAS

Lemma 2.1. ([7]) Let f(z) be a meromorphic function. Then for all irreducible rational functions in f,

$$R(z, f(z)) = \frac{P(z,f(z))}{Q(z,f(z))} = \frac{\sum_{i=0}^{p} a_i(z) f^i}{\sum_{j=0}^{q} b_j(z) f^j},$$

such that the meromorphic coefficients $a_i(z)$, $b_j(z)$satisfy

$T(r, a_i(z)) = S(r, f)$, i = 0, 1, 2,...,p,

$T(r, b_j(z)) = S(r, f)$, j = 0, 1, 2,...,q,

one has$T(r, R(z, f)) = \max\{p, q\} T(r, f) + S(r, f)$.

Lemma 2.2. ([9]) If f(z) is a transcendental meromorphic function, then

$$T(r, f^{(k)}) \leq (k + 1)T(r, f) + S(r, f).$$

Lemma 2.3. ([2]) Let f(z) be a meromorphic function with order $\rho = \rho(f)$, $\rho < +\infty$, and let c be a fix nonzero complex number, then for each $\varepsilon > 0$, one has

$$T(r, f(z + c)) = T(r, f(z)) + O(r^{\rho-1+\varepsilon}) + O(\log r).$$

Lemma 2.4. ([4]) Let f be a transcendental meromorphic function, and $p(z) = a_k z^k + a_{k-1} z^{k-1} + \ldots + a_1 z + a_0$, $a_k \neq 0$ be a nonconstant polynomial of degree k. Given $0 < \delta < |a_k|$, denote $\alpha = |a_k| + \delta$ and $\beta = |a_k| - \delta$. Then given $\varepsilon > 0$ and $a \in \mathbb{C} \cup \{\infty\}$, one has

$$k n(\beta r^k, a, f) \leq n(r, a, f(p(z))) \leq k n(\alpha r^k, a, f)$$

$$N(\beta r^k, a, f) + O(\log r) \leq N(r, a, f(p(z))) \leq N(\alpha r^k, a, f) + O(\log r),$$

$$(1 - \varepsilon) T(\beta r^k, f) \leq T(r, f(p(z))) \leq (1 + \varepsilon) T(\alpha r^k, f)$$

for all r large enough.

Lemma 2.5. ([3]) Let $f : (r_0, +\infty) \to (0, +\infty)$ be positive and bounded in every finite interval, and suppose that $f(\alpha r^k) \leq Af(r) + B$ holds for all r large enough, where $\alpha > 0$, $k > 1$, $A > 1$ and B are real constants. Then

$f(r) = O((\log r)^{\beta})$,

where $\beta = \frac{\log A}{\log k}$.

3. PROOFS OF THE THEOREMS.

In this section we present the proofs of the main results.

Proof of Theorem 1.1. Since the coefficients $\alpha_{ij}(z), \beta_{ij}(z)$ (i = 1, 2; j = 1, 2,..., n), a_{ij}(j = 1, 2,..., p_i; i = 1, 2) and b_{ij}(j = 1, 2,..., q_i; i = 1, 2) in system (1.1) and (1.2) are small functions relative to $f_i(z)$ (i = 1, 2), respectively, that is,

$T(r, \alpha_{ij}) = S(r, f_i)$, i = 1, 2; j = 1, 2,...,n;

$T(r, \beta_{ij}) = S(r, f_i)$, i = 1, 2; j = 1, 2,...,n;

$T(r, a_{ij}) = S(r, f_i)$, i = 1, 2; j = 1, 2,...,p_i;

$T(r, b_{ij}) = S(r, f_i)$, i =1, 2; j = 1, 2,...,q_i

hold for all r outside of a possible exceptional set E_1 with finite logarithmic measure$\int_{E_1} \frac{dr}{r} < +\infty$. Let $(f_1(z), f_2(z))$ be a finite order meromorphic solution of (1.1) and (1.2). Now equating the Nevanlinna characteristic function on both sides of each equation of (1.1) and (1.2), and applying Lemmas 3.1, 3.2 and 3.3, we have

$$d_1 T(r, f_2) = T(r, R_2(z, f_2)) + S(r, f_2)$$

$$= T\left(r, \sum_{j=1}^{n} \left[\alpha_{1j}(z) f_1^{(\lambda_{1j})}(z + c_j) + \beta_{1j}(z) f_1^{m}(z)\right]\right.$$

$$+ S(r, f_2)$$

$$\leq \sum_{j=1}^{n} \left[(\lambda_{1j} + 1) T\left(r, f_1(z + c_j)\right) + mT(r, f_1(z))\right]$$

$$+ S(r, f_2)$$

$$= \sum_{j=1}^{n} [(\lambda_{1j} + m + 1) T(r, f_1(z))] + O(r^{\rho-1+\varepsilon})$$

$$+ O(\log r) + S(r, f_2)$$

$$= (\lambda_1 + m) T(r, f_1(z)) + O(r^{\rho-1+\varepsilon}) + O(\log r) + S(r, f_2).$$

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So there is

(3.1)

$$(d_1 + o(1))\, T(r, f_2) \leq (\lambda_1 + m)T(r, f_1(z)) + O(r^{\rho-1+\varepsilon}) + O(\log r)$$

Similarly, we have

(3.2)

$$(d_2 + o(1))T(r, f_1) \leq (\lambda_2 + m)T(r, f_2(z)) + O(r^{\rho-1+\varepsilon}) + O(\log r)$$

Thus, (3.1) times (d_2 + o(1)) and (3.2) times (λ_1+ m) obtain respectively that

(3.3)

$$(d_2+o(1))(d_1+o(1))T(r, f_2) \leq (\lambda_1+m)T(r, f_1)(d_2+o(1)) + (d_2+o(1))[O(r^{\rho-1+\varepsilon}) + O(\log r)],$$

and

(3.4)

$$(\lambda_1+m)(d_2+o(1))T(r, f_1) \leq (\lambda_1+m)(\lambda_2+m)T(r, f_2)+(\lambda_1+m)[O(r^{\rho-1+\varepsilon}) + O(\log r)]$$

Therefore, from (3.3) and (3.4), we can have the following inequality

$$(d_2 + o(1))(d_1 + o(1))T(r, f_2) \leq (\lambda_1+m)(\lambda_2+m)\, T(r, f_2) + (\lambda_1 + m + (d_2 + o(1)))[O(r^{\rho-1+\varepsilon}) + O(\log r)] + S(r, f_2).$$

Dividing this by T (r, f_2) on both sides of the last inequality and letting r → ∞ outside of a possible exceptional set E_1 with finite logarithmic measure respectively, we have $d_1 d_2 \leq (\lambda_1+m)(\lambda_2+m)$.Therefore, the proof of Theorem 1.1 is completed.

Proof of Theorem 1.2. Let ($f_1(z)$, $f_2(z)$) be a transcendental meromorphic solution of system (1.3) and (1.4), and $p(z) = a_k z^k + a_{k-1} z^{k-1} + \ldots + a_1 z + a_0, a_k \neq 0$ be a nonconstant polynomial of degree k. Given $0 < \delta < |a_k|$, denote $\alpha = |a_k| + \delta$ and $\beta = |a_k| - \delta$. Denoting again C = max $\{|c_i|$, i = 1, 2,..., n$\}$. According to system (1.3) and (1.4) and the last result of Lemma 2.4, for given $\varepsilon_1 > 0$, we have

$$(1-\varepsilon_1)T(\beta r^k, f_1) \leq T(r, f_1(p(z)))$$

$$= T(r, \textstyle\sum_{j=1}^{n} [\alpha_{2j}(z) f_2^{(\lambda_{2j})}(z + c_j) + \beta_{2j}(z) f_2^m(z)] + S(r, f_2)$$

$$\leq \sum_{j=1}^{n} [T(r, f_2^{(\lambda_{2j})}(z + c_j)) + T(r, f_2^m(z))] + S(r, f_2)$$

$$\leq \sum_{j=1}^{n} \left[(\lambda_{2j} + 1)T\left(r, f_2(z + c_j)\right) + mT\left(r, f_2(z)\right)\right] + S(r, f_2)$$

$$\leq (\lambda_2 + m)\, T(r + C, f_2(z)) + S(r, f_2)$$

Since T (r + C, f) ≤ T (β r, f) holds for r large enough for $\beta > 1$, we may assume r to be large enough to satisfy

$$(1-\varepsilon_1)T(\beta r^k, f_1) \leq (\lambda_2 + m)(1 + \varepsilon)T(\beta r, f_2),$$

outside a possible exceptional set of nite linear measure. By the standard reasoning to remove the exceptional set, we know that whenever $\mu > 1$,

(3.5) $$(1-\varepsilon_1)T(\beta r^k, f_1) \leq (\lambda_2 + m)(1 + \varepsilon_1)T(\mu\beta r, f_2),$$

holds for all r large enough. Similarly, we also obtain

(3.6) $$(1-\varepsilon_2)T(\beta r^k, f_2) \leq (\lambda_1 + m)(1 + \varepsilon_2)T(\mu\beta r, f_1)$$

holds for all r large enough. Taking $\varepsilon = \max\{\varepsilon_1, \varepsilon_2\}$; $\lambda = \max\{\lambda_1, \lambda_2\}$, and (3.5)+(3.6) assumes that

$$(1-\varepsilon)[T(\beta r^k, f_1) + T(\beta r^k, f_2)] \leq (+ m)(1 + \varepsilon)[T(\mu\beta r, f_1) + T(\mu\beta r, f_2)].$$

Denote t = $\mu\beta$r, thus the last inequality may be written in the form

$$\left[T\left(\frac{\beta}{(\mu\beta)^k} t^k, f_1\right) + T\left(\frac{\beta}{(\mu\beta)^k} t^k, f_2\right)\right] \leq \frac{(\lambda + m)(1 + \varepsilon)}{(1 - \varepsilon)}[T(t, f_1) + T(t, f_2)].$$

Therefore, by Lemma 2.5 and (3.7) we can get

$$T(r, f_1) + T(r, f_2) = O((\log r)^{\gamma})$$

where$\gamma=\frac{\log(\lambda(1+\varepsilon)/(1-\varepsilon))}{\log k}=\frac{\log\lambda}{\log k}+o(1)$. Now denoting $\alpha=\frac{\log\lambda}{\log k}$, we can obtain the required form of Theorem 1.2 . Therefore, Theorem 1.2 is proved.

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